

Spectator Scattering and Annihilation Contributions as a Solution to the πK and $\pi\pi$ Puzzles within QCD Factorization Approach

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Abstract

The large branching ratios for pure annihilation $\bar{B}_s^0 \rightarrow \pi^+\pi^-$ and $\bar{B}_d^0 \rightarrow K^+K^-$ decays reported by CDF and LHCb collaborations recently and the so-called πK and $\pi\pi$ puzzles indicate that spectator scattering and annihilation contributions are important to the penguin-dominated, color-suppressed tree dominated, and pure annihilation nonleptonic B decays. Combining the available experimental data for $B_{u,d} \rightarrow \pi\pi$, πK and $K\bar{K}$ decays, we do a global fit on the spectator scattering and annihilation parameters $X_H(\rho_H, \phi_H)$, $X_A^i(\rho_A^i, \phi_A^i)$ and $X_A^f(\rho_A^f, \phi_A^f)$, which are used to parameterize the endpoint singularity in amplitudes of spectator scattering, nonfactorizable and factorizable annihilation topologies within the QCD factorization framework, in three scenarios for different purpose. Numerically, in scenario II, we get $(\rho_A^i, \phi_A^i[^\circ]) = (2.88_{-1.30}^{+1.52}, -103_{-40}^{+33})$ and $(\rho_A^f, \phi_A^f[^\circ]) = (1.21_{-0.25}^{+0.22}, -40_{-8}^{+12})$ at the 68% confidence level, which are mainly demanded by resolving πK puzzle and confirm the presupposition that $X_A^i \neq X_A^f$. In addition, correspondingly, the B -meson wave function parameter λ_B is also fitted to be $0.18_{-0.08}^{+0.11} MeV$, which plays an important role for resolving both πK and $\pi\pi$ puzzles. With the fitted parameters, the QCDF results for observables of $B_{u,d} \rightarrow \pi\pi$, πK and $K\bar{K}$ decays are in good agreement with experimental measurements. Much more experimental and theoretical efforts are expected to understand the underlying QCD dynamics of spectator scattering and annihilation contributions.

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I. INTRODUCTION

Charmless hadronic B -meson decays provide a fertile ground for testing the Standard Model (SM) and exploring the source of CP violation, which attract much attention in the past years. Thanks to the fruitful accomplishment of BABAR and Belle, the constraints on the sides and interior angles of the unitarity triangle significantly reduce the allowed ranges of some of the CKM elements, and many rare B decays are well measured. With the successful running of LHC and the advent of Belle II at SuperKEKB, heavy flavour physics has entered a new exciting era and more B decay modes will be measured precisely soon.

Recently, the evidence of pure annihilation decays $\bar{B}_s^0 \rightarrow \pi^+\pi^-$ and $\bar{B}_d^0 \rightarrow K^+K^-$ are firstly reported by CDF Collaboration [1], and soon confirmed by LHCb Collaboration [2]. The Heavy Flavor Averaging Group (HFAG) presents their branching ratios [3]

$$\mathcal{B}(\bar{B}_s^0 \rightarrow \pi^+\pi^-) = (0.73 \pm 0.14) \times 10^{-6}, \quad (1)$$

$$\mathcal{B}(\bar{B}_d^0 \rightarrow K^+K^-) = (0.12 \pm 0.05) \times 10^{-6}. \quad (2)$$

Such results, if confirmed, imply unexpectedly large annihilation contributions in B decays and significant flavour symmetry breaking effects between the annihilation amplitudes of $B_{u,d}$ and B_s decays, which attract much attention recently, for instance Refs. [4–7].

Theoretically, as noticed already in Refs. [8–11], even though the annihilation contributions are formally Λ_{QCD}/m_b power suppressed, they are very important and indispensable for charmless B decays. By introducing the parton transverse momentum and the Sudakov factor to regulate the endpoint divergence, there is a large complex annihilation contribution within the perturbative QCD (pQCD) approach [8, 9]. The latest renewed pQCD estimations¹ $\mathcal{B}(\bar{B}_s^0 \rightarrow \pi^+\pi^-) = (5.10_{-1.68-0.19-0.83-0.20}^{+1.96+0.25+1.05+0.29}) \times 10^{-7}$ and $\mathcal{B}(\bar{B}_d^0 \rightarrow K^+K^-) = (1.56_{-0.42-0.22-0.19-0.09}^{+0.44+0.23+0.22+0.13}) \times 10^{-7}$ [7] give an appropriate account of the CDF and LHCb measurements within uncertainties. In the QCD factorization (QCDF) framework [12], the endpoint divergence in annihilation amplitudes is usually parameterized by $X_A(\rho_A, \phi_A)$ (see Eq.(9)). The parameters $\rho_A \sim 1$ and $\phi_A \sim -55^\circ$ (scenario S4) [11] are adopted conservatively in evaluating the amplitudes of $B \rightarrow PP$ decays, which lead to the predictions² $\mathcal{B}(\bar{B}_s^0 \rightarrow \pi^+\pi^-) = (0.26_{-0.00-0.09}^{+0.00+0.10}) \times 10^{-6}$ and $\mathcal{B}(\bar{B}_d^0 \rightarrow K^+K^-) = (0.10_{-0.02-0.03}^{+0.03+0.03}) \times 10^{-6}$ [13].

¹ The first three uncertainties come from meson wave functions, the last one is from the CKM factors.

² The second uncertainty comes from parameters $\rho_{A,H}$ and $\phi_{A,H}$ of annihilation and spectator contributions.

It is obvious that the QCDF prediction of $\mathcal{B}(\bar{B}_d^0 \rightarrow K^+ K^-)$ agrees well with the data Eq.(2), but the one of $\mathcal{B}(\bar{B}_s^0 \rightarrow \pi^+ \pi^-)$ is much smaller than the present experimental measurement Eq.(1). This discrepancy kindles the passions of restudy on annihilation contributions [4–6].

At present, there are two major issues among the well-concerning focus on the annihilation contributions within the QCDF framework, one is whether $X_A(\rho_A, \phi_A)$ is universal for B decays, and the other is what its value should be. As to the first issue, there is no an imperative reason for the annihilation parameters ρ_A and ϕ_A to be the same for different $B_{u,d,s}$ decays, even for different annihilation topologies, although they were usually taken to be universal in the previous numerical calculation for simplicity [10, 11]. Phenomenologically, it is almost impossible to account for all of the well-measured two-body charmless B decays with the universal values of ρ_A and ϕ_A based on the QCDF approach [5, 6, 11, 13]. In addition, the pQCD study on B meson decays also indicate that the annihilation parameters ρ_A and ϕ_A should be process-dependent. In fact, in the practical QCDF application to the $B \rightarrow PP$, PV decays (where P and V denote the light pseudoscalar and vector $SU(3)$ meson nonet, respectively), the non-universal values of annihilation phase ϕ_A with respect to PP and PV final states are favored (scenario S4) [11]; the process-dependent values of ρ_A and ϕ_A are given based on an educated guess [13, 14] or the comparison with the updated measurements [6]; the flavour-dependent values of ρ_A and ϕ_A are suggested recently in the nonfactorizable annihilation contributions [5]. In principle the value of ρ_A and ϕ_A should differ from each other for different topologies with different flavours, but we hope that the QCDF approach can accommodate and predict much more hadronic B decays with less input parameters. So much attention in phenomenological analysis on the weak annihilation B decays is devoted to what the appropriate values of the parameters ρ_A and ϕ_A should be. This is the second issue. In principle, a large value of ρ_A is unexpected by the power counting rules and the self-consistency validation within the QCDF framework. The original proposal is that $\rho_A \leq 1$ and an arbitrary strong interaction phase ϕ_A are universal for all decay processes, and that a fine-tuning of the phase ϕ_A is required to be reconciled with experimental data when ρ_A is significantly larger than 1 [11]. The recent study on the annihilation contributions show that $\rho_A > 2$ and $|\phi_A| \geq 30^\circ$ are acceptable, even necessary, to reproduce the data for some two-body nonleptonic $B_{u,d,s}$ decay modes [5, 6]. In this paper, we will perform a fitting on the parameters ρ_A and ϕ_A by considering $B \rightarrow \pi\pi$, πK and $K\bar{K}$ decay modes, on one hand, to investigate the strength of annihilation contribution, on

the other hand, to study their effects on the anomalies in B physics, such as the well-known πK and $\pi\pi$ puzzles.

The so-called πK puzzle is reflected by the difference between the direct CP asymmetries for $B^- \rightarrow K^- \pi^0$ and $\bar{B}^0 \rightarrow K^- \pi^+$ decays. With the up-to-date HFAG results [3], we get

$$\Delta A \equiv A_{CP}(B^- \rightarrow K^- \pi^0) - A_{CP}(\bar{B}^0 \rightarrow K^- \pi^+) = (12.2 \pm 2.2)\%, \quad (3)$$

which differs from zero by about 5.5σ . However, the direct CP asymmetries of $A_{CP}(B^- \rightarrow K^- \pi^0)$ and $A_{CP}(\bar{B}^0 \rightarrow K^- \pi^+)$ are expected to be approximately equal with the isospin symmetry in the SM, numerically for instance $\Delta A \sim 0.5\%$ in the S4 scenario of QCDF [11].

The so-called $\pi\pi$ puzzle is reflected by the following two ratios of the CP -averaged branching fractions [15]:

$$R_{+-}^{\pi\pi} \equiv 2 \left[\frac{\mathcal{B}(B^- \rightarrow \pi^- \pi^0)}{\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \pi^-)} \right] \frac{\tau_{B^0}}{\tau_{B^+}}, \quad R_{00}^{\pi\pi} \equiv 2 \left[\frac{\mathcal{B}(\bar{B}^0 \rightarrow \pi^0 \pi^0)}{\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \pi^-)} \right]. \quad (4)$$

It is generally expected that branching ratio $\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \pi^-) \gtrsim \mathcal{B}(B^- \rightarrow \pi^- \pi^0)$ and $\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \pi^-) \gg \mathcal{B}(\bar{B}^0 \rightarrow \pi^0 \pi^0)$ within the SM. To date, the agreement of $R_{+-}^{\pi\pi}$ between the S4 scenario QCDF $R_{+-}^{\pi\pi}(\text{QCDF}) = 1.83$ [11] and the refined experimental data $R_{+-}^{\pi\pi}(\text{Exp.}) = 1.99 \pm 0.15$ [3] can be achieved consistently within experimental error, while the discrepancy in $R_{00}^{\pi\pi}$ between the S4 scenario QCDF $R_{00}^{\pi\pi}(\text{QCDF}) = 0.27$ (where theoretical uncertainties are unenclosed) [11] and the progressive experimental data $R_{+-}^{\pi\pi}(\text{Exp.}) = 1.99 \pm 0.15$ [3] is unexpectedly large.

It is claimed [15] that the so-called $\pi\pi$ puzzle could be accommodated by the nonfactorizable contributions in SM. It is argued [14, 15] that to solve the so-called πK puzzle, a large complex color-suppressed tree amplitude C' or a large complex electroweak penguin contribution P'_{EW} or a combination of them are essential. An enhanced complex P'_{EW} with a nontrivial strong phase can be obtained from new physics effects [15]. To get a large complex C' , one can resort to spectator scattering and final state interactions [13, 14]. Recently, the annihilation amplitudes with large parameters ρ_A is suggested to conciliate the recent measurements Eq.(1) and Eq.(2), so surprisingly, the πK puzzle is also resolved simultaneously [5]. Theoretically, the power corrections, such as spectator scattering at the twist-3 order and annihilation amplitudes, are important to account for the large branching ratios and CP asymmetries of penguin-dominated and/or color-suppressed tree-dominated B decays. So, before claiming a new physics signal, it is essential to examine whether power

corrections could retrieve “problematic” deviations from the SM expectations. Interestingly, our study show that with appropriate parameters, the annihilation and spectator scattering contributions could provide some possible solutions to the πK and $\pi\pi$ puzzles.

Our paper is organized as following. In section II, we give a brief overview of the hard spectator and annihilation calculations and recent studies within QCDF. In section III, focusing on πK and $\pi\pi$ puzzles, the effects of spectator scattering and annihilation contributions on $B \rightarrow \pi\pi$, πK and $K\bar{K}$ decays are studied in detail in three scenarios. In each scenario, a fitting on relevant parameters are performed. Our conclusions are summarized in section IV. Appendix A recapitulates the building blocks of annihilation and spectator scattering amplitudes. The input parameters and our fitting approach are given in Appendix B and C, respectively.

II. BRIEF REVIEW OF SPECTATOR SCATTERING AND ANNIHILATION AMPLITUDES WITHIN QCDF

The effective Hamiltonian for nonleptonic B weak decays is [16]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p,q} V_{pb} V_{pq}^* \left\{ \sum_{i=1}^{10} C_i O_i + C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} \right\} + \text{h.c.}, \quad (5)$$

where $V_{pb} V_{pq}^*$ ($p = u, c$ and $q = d, s$) is the product of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements; C_i is the Wilson coefficient corresponding to the local four-quark operator O_i ; $O_{7\gamma}$ and O_{8g} are the electromagnetic and chromomagnetic dipole operators.

With the effective Hamiltonian Eq.(5), the QCDF method has been fully developed and extensively employed to calculate the hadronic two-body B decays, for example, see [10–13]. The spectator scattering and annihilation amplitudes (see Fig.1) are expressed as the convolution of scattering functions with the light-cone wave functions of the participating mesons [11, 12]. The explicit expressions for the basic building blocks of spectator scattering and annihilation amplitudes have been given by Ref. [11], which are also listed in the appendix A for convenience. With the asymptotic light-cone distribution amplitudes, the building blocks for annihilation amplitudes of Eq.(A1-A5) could be simplified as [11]

$$A_1^i \simeq A_2^i \simeq 2\pi\alpha_s \left[9 \left(X_A - 4 + \frac{\pi^2}{3} \right) + r_\chi^{M_1} r_\chi^{M_2} X_A^2 \right], \quad (6)$$

$$A_3^i \simeq 6\pi\alpha_s (r_\chi^{M_1} - r_\chi^{M_2}) \left(X_A^2 - 2X_A + \frac{\pi^2}{3} \right), \quad (7)$$

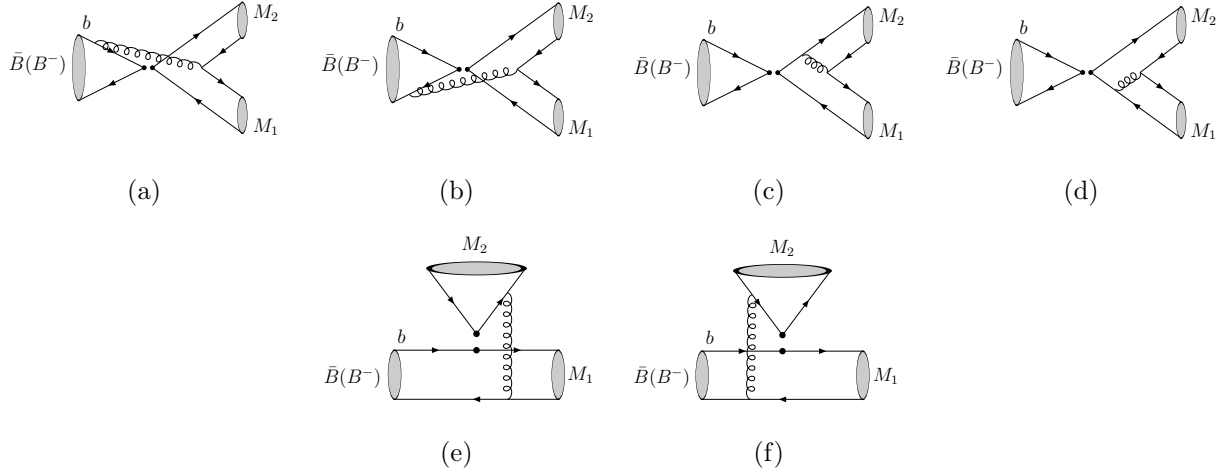


FIG. 1: The lowest order diagrams of weak annihilation (a-d) and spectator scattering (e,f).

$$A_3^f \simeq 6\pi\alpha_s(r_\chi^{M_1} + r_\chi^{M_2})(2X_A^2 - X_A), \quad (8)$$

where the superscripts i (or f) refers to gluon emission from the initial (or final) state quarks, respectively (see Fig.1). For the $\pi\pi$, πK and $K\bar{K}$ final-state, A_3^i is numerically negligible due to $r_\chi^{M_1} \simeq r_\chi^{M_2}$. The model-dependent parameter X_A is used to estimate the endpoint contributions, and expressed as

$$\int_0^1 \frac{dx}{x} \rightarrow X_A = (1 + \rho_A e^{i\phi_A}) \ln \frac{m_B}{\Lambda_h}, \quad (9)$$

where $\Lambda_h = 0.5$ GeV. For spectator scattering contributions, the calculation of twist-3 distribution amplitudes also suffers from endpoint divergence, which is usually dealt with the same manner as Eq.(9) and labelled by X_H [11]. Moreover, a quantity λ_B is used to parameterize our ignorance about B -meson distribution amplitude [see Eq.(A6)] through [11]

$$\int_0^1 \frac{d\xi}{\xi} \Phi_B(\xi) \equiv \frac{m_B}{\lambda_B}. \quad (10)$$

The QCDF approach itself cannot give information or/and constraint on the phenomenological parameters of X_A , X_H and λ_B . These parameters should be determined from experimental data. To conform with measurements of nonleptonic $B \rightarrow PP$ decays, we will adopt a similar method used in Ref.[5] to deal with the contributions from weak annihilation and spectator scattering. Focusing on the flavor dependence, without consideration of theoretical uncertainties, annihilation contributions are reevaluated in detail [5] to explain the πK puzzle and the recent measurements on pure annihilation decays $\bar{B}_s^0 \rightarrow \pi^+\pi^-$ and \bar{B}_d^0

$\rightarrow K^+K^-$ [see Eq.(1,2)]. The authors of Ref. [5] find that the flavour symmetry breaking effects should be carefully considered for $B_{u,d,s}$ decays, and suggest that the parameters of ρ_A and ϕ_A in nonfactorizable annihilation topologies A_k^i [see Eq.(6,7)] should be different from those in factorizable annihilation topologies A_k^f [see Eq.(8)]. (1) For factorizable annihilation topologies, i.e., the gluon emission from the final states Fig.1(c,d), the flavor symmetry breaking effects are embodied in the decay constants, because the asymptotic light-cone distribution amplitudes of final states are the same. In addition, all decay constants have been factorized outside from the hadronic matrix elements of factorizable annihilation topologies. So A_k^f is independent of the initial state, and is the same for $B_{u,d,s}$ annihilation decays to two light pseudoscalar mesons, that is to say, ρ_A^f and ϕ_A^f should be universal for $B_{u,d,s} \rightarrow PP$ decays. (2) For nonfactorizable annihilation topologies, i.e., the gluon emission from the initial B meson Fig.1(a,b), besides the factorized decay constants and the same asymptotic light-cone distribution amplitudes, B meson wave functions $\Phi_B(\xi)$ are involved in the convolution integrals of hadronic matrix elements. Hence, A_k^i should depend on the initial state and be different for $B_{u,d}$ from B_s mesons due to flavor symmetry breaking effects, i.e., parameters of ρ_A^i and ϕ_A^i should be non-universal for B_s and $B_{u,d}$ meson decays, and be different from parameters of ρ_A^f and ϕ_A^f for A_k^f . In fact, the symmetry breaking effects have been considered in previous QCDF study on two-body hadronic B decays [6, 11, 13, 14, 17], but with parameters of $\rho_A^f = \rho_A^i$ and $\phi_A^f = \phi_A^i$. So, it is essential to systematically reevaluate factorizable and nonfactorizable annihilation contributions and perform a global fit on the annihilation parameters with the current available experimental data. In this paper, we will pay much attention to $B_{u,d} \rightarrow KK, \pi K, \pi\pi$ decays and the aforementioned $\pi K, \pi\pi$ puzzles with a distinction between (ρ_A^f, ϕ_A^f) and (ρ_A^i, ϕ_A^i) , i.e., $X_A^i \neq X_A^f$.

As aforesaid [14, 15], the nonfactorizable spectator scattering amplitudes contribute to a large complex C' , which is important to resolve the $\pi K, \pi\pi$ puzzles. From the building block Eq.(A6), it can be easily seen that B meson wave functions $\Phi_B(\xi)$ appear in the spectator scattering amplitudes. Therefore, the symmetry breaking effects should also be considered for the quantity X_H that is introduced to parameterize the endpoint singularity in the twist-3 level spectator scattering corrections. Similar to X_A^i , the quantity X_H is related to the topologies that gluon emit from the initial B meson. So, for simplicity, the approximation $X_H = X_A^i$ is assumed in our coming numerical evaluation (scenarios I and II, see the next section for detail). Of course, this approximation is neither based on solid

ground or from some underlying principle, and should be carefully studied and deserve much research. In fact, our coming phenomenological study (scenarios III) shows that the approximation $X_H = X_A^i$ is allowable with the up-to-date measurement on $B_{u,d} \rightarrow KK, \pi K, \pi\pi$ decays. In addition, it can be seen from Eq.(A6) that the spectator scattering corrections depend strongly on the inverse moment parameter λ_B given in Eq.(10). Recently, the value of λ_B is an increasing concern of theoretical and experimental physicists [18–23]. A scrutiny of parameter λ_B becomes imperative. In this paper, we will give some information on λ_B required by present experimental data of $B_{u,d} \rightarrow K\bar{K}, \pi K, \pi\pi$ decays.

III. NUMERICAL ANALYSIS AND DISCUSSIONS

With the conventions in Ref. [11], the decay amplitudes for $B_{u,d} \rightarrow \pi K, K\bar{K}, \pi\pi$ decays within the QCDF framework can be written as

$$\mathcal{A}_{B^- \rightarrow \pi^- \bar{K}^0} = \sum_{p=u,c} V_{pb} V_{ps}^* A_{\pi K} \left\{ \alpha_4^p - \frac{1}{2} \alpha_{4,\text{EW}}^p + \delta_{pu} \beta_2 + \beta_3^p + \beta_{3,\text{EW}}^p \right\}, \quad (11)$$

$$\begin{aligned} \sqrt{2} \mathcal{A}_{B^- \rightarrow \pi^0 K^-} &= \sum_{p=u,c} V_{pb} V_{ps}^* \left\{ A_{\pi K} \left[\delta_{pu} (\alpha_1 + \beta_2) + \alpha_4^p + \alpha_{4,\text{EW}}^p + \beta_3^p + \beta_{3,\text{EW}}^p \right] \right. \\ &\quad \left. + A_{K\pi} \left[\delta_{pu} \alpha_2 + \frac{3}{2} \alpha_{3,\text{EW}}^p \right] \right\}, \end{aligned} \quad (12)$$

$$\mathcal{A}_{\bar{B}^0 \rightarrow \pi^+ K^-} = \sum_{p=u,c} V_{pb} V_{ps}^* A_{\pi K} \left\{ \delta_{pu} \alpha_1 + \alpha_4^p + \alpha_{4,\text{EW}}^p + \beta_3^p - \frac{1}{2} \beta_{3,\text{EW}}^p \right\}, \quad (13)$$

$$\begin{aligned} \sqrt{2} \mathcal{A}_{\bar{B}^0 \rightarrow \pi^0 \bar{K}^0} &= \sum_{p=u,c} V_{pb} V_{ps}^* \left\{ A_{\pi K} \left[-\alpha_4^p + \frac{1}{2} \alpha_{4,\text{EW}}^p - \beta_3^p + \frac{1}{2} \beta_{3,\text{EW}}^p \right] \right. \\ &\quad \left. + A_{K\pi} \left[\delta_{pu} \alpha_2 + \frac{3}{2} \alpha_{3,\text{EW}}^p \right] \right\}, \end{aligned} \quad (14)$$

$$\mathcal{A}_{B^- \rightarrow K^0 \bar{K}^0} = \sum_{p=u,c} V_{pb} V_{pd}^* A_{KK} \left\{ \alpha_4^p - \frac{1}{2} \alpha_{4,\text{EW}}^p + \delta_{pu} \beta_2 + \beta_3^p + \beta_{3,\text{EW}}^p \right\}, \quad (15)$$

$$\mathcal{A}_{\bar{B}^0 \rightarrow K^- K^+} = \sum_{p=u,c} V_{pb} V_{pd}^* \left\{ B_{\bar{K}K} \left[\delta_{pu} b_1 + b_4^p + b_{4,\text{EW}}^p \right] + B_{K\bar{K}} \left[b_4^p - \frac{1}{2} b_{4,\text{EW}}^p \right] \right\}, \quad (16)$$

$$\begin{aligned} \mathcal{A}_{\bar{B}^0 \rightarrow \bar{K}^0 K^0} &= \sum_{p=u,c} V_{pb} V_{pd}^* \left\{ A_{\bar{K}K} \left[\alpha_4^p - \frac{1}{2} \alpha_{4,\text{EW}}^p + \beta_3^p + \beta_4^p - \frac{1}{2} \beta_{3,\text{EW}}^p - \frac{1}{2} \beta_{4,\text{EW}}^p \right] \right. \\ &\quad \left. + B_{K\bar{K}} \left[b_4^p - \frac{1}{2} b_{4,\text{EW}}^p \right] \right\}, \end{aligned} \quad (17)$$

$$\sqrt{2} \mathcal{A}_{B^- \rightarrow \pi^- \pi^0} = \sum_{p=u,c} V_{pb} V_{pd}^* A_{\pi\pi} \left\{ \delta_{pu} (\alpha_1 + \alpha_2) + \frac{3}{2} (\alpha_{3,\text{EW}}^p + \alpha_{4,\text{EW}}^p) \right\}, \quad (18)$$

$$\begin{aligned} \mathcal{A}_{\bar{B}^0 \rightarrow \pi^+ \pi^-} &= \sum_{p=u,c} V_{pb} V_{pd}^* A_{\pi\pi} \left\{ \delta_{pu} (\alpha_1 + \beta_1) + \alpha_4^p + \alpha_{4,\text{EW}}^p + \beta_3^p + 2\beta_4^p \right. \\ &\quad \left. - \frac{1}{2} \beta_{3,\text{EW}}^p + \frac{1}{2} \beta_{4,\text{EW}}^p \right\}, \end{aligned} \quad (19)$$

$$\begin{aligned}
-\mathcal{A}_{\bar{B}^0 \rightarrow \pi^0 \pi^0} = & \sum_{p=u,c} V_{pb} V_{pd}^* A_{\pi\pi} \left\{ \delta_{pu} (\alpha_2 - \beta_1) - \alpha_4^p + \frac{3}{2} \alpha_{3,\text{EW}}^p + \frac{1}{2} \alpha_{4,\text{EW}}^p \right. \\
& \left. - \beta_3^p - 2\beta_4^p + \frac{1}{2} \beta_{3,\text{EW}}^p - \frac{1}{2} \beta_{4,\text{EW}}^p \right\}.
\end{aligned} \tag{20}$$

For the sake for convenient discussion, we reiterate the expressions of the annihilation coefficients [11],

$$\beta_i^p = b_i^p B_{M_1 M_2} / A_{M_1 M_2}, \tag{21}$$

$$b_1 = \frac{C_F}{N_c^2} C_1 A_1^i, \quad b_2 = \frac{C_F}{N_c^2} C_2 A_1^i, \tag{22}$$

$$b_3^p = \frac{C_F}{N_c^2} [C_3 A_1^i + C_5 (A_3^i + A_3^f) + N_c C_6 A_3^f], \tag{23}$$

$$b_4^p = \frac{C_F}{N_c^2} [C_4 A_1^i + C_6 A_2^i], \tag{24}$$

$$b_{3,\text{EW}}^p = \frac{C_F}{N_c^2} [C_9 A_1^i + C_7 (A_3^i + A_3^f) + N_c C_8 A_3^f], \tag{25}$$

$$b_{4,\text{EW}}^p = \frac{C_F}{N_c^2} [C_{10} A_1^i + C_8 A_2^i]. \tag{26}$$

Numerically, coefficients of $b_{3,\text{EW}}^p$ and $b_{4,\text{EW}}^p$ are negligible compared with the other effective coefficients due to the small electroweak Wilson coefficients, and so their effects would be not discussed in this paper.

In order to illustrate the contributions of annihilation and spectator scattering, we explore three parameter scenarios in which certain parameters are changed freely.

- Scenario I: $B_{u,d} \rightarrow \pi K$ and $K \bar{K}$ decays, including the πK puzzle and pure annihilation decay $B_d \rightarrow K^- K^+$, are studied in detail. Combining the latest experimental data on the CP -averaged branching ratios, direct and mixing-induced CP -asymmetries, total 14 observables (see Table.II, III, IV) for seven $B_{u,d} \rightarrow \pi K$, $K \bar{K}$ decay modes [see Eq.(11—17)], the fit on four parameters (ρ_A^f, ϕ_A^f) and (ρ_A^i, ϕ_A^i) is performed with the fixed value $\lambda_B = 0.2$ GeV and the approximation $(\rho_H, \phi_H) = (\rho_A^i, \phi_A^i)$, where (ρ_A^f, ϕ_A^f) , (ρ_A^i, ϕ_A^i) and (ρ_H, ϕ_H) are assumed to be universal for factorizable annihilation amplitudes, nonfactorizable annihilation amplitudes and spectator scattering corrections, respectively.
- Scenario II: $B_{u,d} \rightarrow \pi K$, $K \bar{K}$ and $\pi\pi$ decays, including $\pi\pi$ puzzle, are studied. Combining the latest experimental data on the CP -averaged branching ratios, direct and mixing-induced CP -asymmetries, total 21 observables (see Table.II, III, IV) for ten

$B_{u,d} \rightarrow \pi K, K\bar{K}, \pi\pi$ decay modes [see Eq.(11—20)], the fit on five parameters (ρ_A^f, ϕ_A^f) , (ρ_A^i, ϕ_A^i) and λ_B is performed with the approximation $(\rho_H, \phi_H) = (\rho_A^i, \phi_A^i)$.

- Scenario III: As a general scenario, to clarify the relative strength among (ρ_A^f, ϕ_A^f) , (ρ_A^i, ϕ_A^i) and (ρ_H, ϕ_H) , and check whether the approximation $(\rho_H, \phi_H) = (\rho_A^i, \phi_A^i)$ is allowed or not, a fit on such six free parameters is performed.

Other input parameters used in our evaluation are summarized in Appendix B. Our fit approach is illustrated in detail in Appendix C.

A. Scenario I

Comparing Eq.(12) with Eq.(13), it can be clearly seen that $\sqrt{2}\mathcal{A}_{B^- \rightarrow \pi^0 K^-} \simeq \mathcal{A}_{\bar{B}^0 \rightarrow \pi^+ K^-}$ if $\delta_{pu}\alpha_2 + \frac{3}{2}\alpha_{3,\text{EW}}^p$ is negligible compared with $\delta_{pu}\alpha_1 + \alpha_4^p$. Hence it is expected $\Delta A \simeq 0$ in SM, which significantly disagrees with the current experimental data in Eq.(3), this is the so-called πK puzzle. To resolve the πK puzzle, one possible solution is that there is a large complex contributions from $\delta_{pu}\alpha_2 + \frac{3}{2}\alpha_{3,\text{EW}}^p$. Many proposals have been offered, such as the enhancement of color-suppressed tree amplitude α_2 in Ref.[14], significant new physics corrections to the electroweak penguin coefficient $\alpha_{3,\text{EW}}^p$ in Ref.[15], and so on. Indeed, it has been shown [11] that the coefficients α_2 and $\alpha_{3,\text{EW}}^p$ are seriously affected by spectator scattering corrections within QCDF framework. Consequently, the nonfactorizable spectator scattering parameters X_H or (ρ_H, ϕ_H) will have great influence on the observable ΔA . Furthermore, a scrutiny of difference between Eq.(12) and Eq.(13), another possible resolution to the πK puzzle might be provided by annihilation contributions, such as coefficient β_2 , as suggested in Ref.[5]. If so, then ΔA will depend strongly on the nonfactorizable annihilation parameters (ρ_A^i, ϕ_A^i) because β_2 is proportional to A_1^i in Eq.(22). Additionally, it can be seen from Eq.(12) and Eq.(13) that annihilation coefficient β_3^p contributes to amplitudes both $\mathcal{A}_{B^- \rightarrow \pi^0 K^-}$ and $\mathcal{A}_{\bar{B}^0 \rightarrow \pi^+ K^-}$. If β_3^p could offer a large strong phase, then its effect should contribute to the direct CP asymmetries $A_{CP}(B^- \rightarrow \pi^0 K^-)$ and $A_{CP}(\bar{B}^0 \rightarrow \pi^+ K^-)$ rather than ΔA . Due to the fact that the lion's share of β_3^p comes from $N_c C_6 A_3^f$ in Eq.(23), the direct CP asymmetries $A_{CP}(B^- \rightarrow \pi^0 K^-)$ and $A_{CP}(\bar{B}^0 \rightarrow \pi^+ K^-)$ should vary greatly with the factorizable annihilation parameters X_A^f , while ΔA should be insensitive to variation of parameters (ρ_A^f, ϕ_A^f) . The above analysis and speculations are confirmed by Fig.2.

From Eq.(16), it is seen that the amplitude $\mathcal{A}_{\bar{B}^0 \rightarrow K^- K^+}$ depends heavily on coefficients β_1 and β_4^p , which are closely associated with the nonfactorizable annihilation parameter X_A^i only. The factorizable annihilation contributions vanish due to the isospin symmetry, which is consistent with the pQCD calculation [7]. The large branching ratio Eq.(2) would appeal for large nonfactorizable annihilation parameter X_A^i or ρ_A^i . The dependence of branching ratio $\mathcal{B}(\bar{B}^0 \rightarrow K^- K^+)$ on the parameters (ρ_A^i, ϕ_A^i) is displayed in Fig.3.

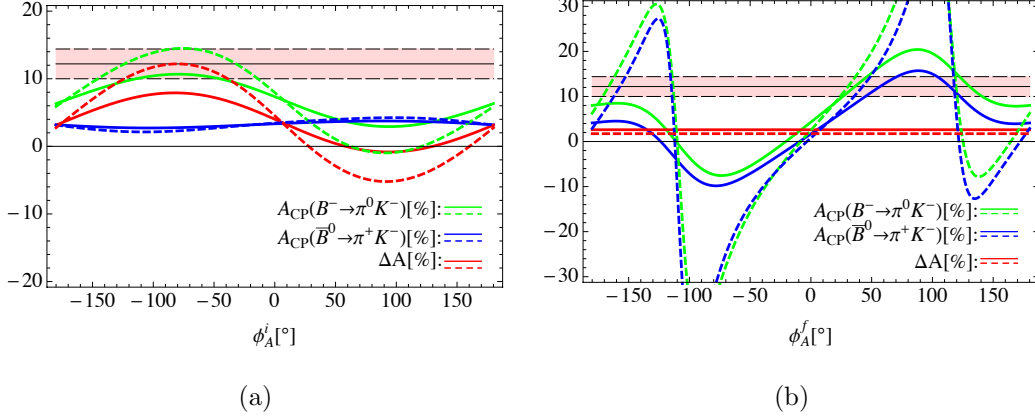


FIG. 2: The direct CP asymmetries $A_{CP}(B^- \rightarrow \pi^0 K^-)$, $A_{CP}(\bar{B}^0 \rightarrow \pi^+ K^-)$ and their difference ΔA via (a) parameters (ρ_A^i, ϕ_A^i) with $\rho_A^f = \phi_A^f = 0$; and (b) parameters (ρ_A^f, ϕ_A^f) with $\rho_A^i = \phi_A^i = 0$, where the solid and dashed lines correspond to $\rho_A^{i,f} = 1$ and 2, respectively; The shaded band is the experimental result for ΔA with 1σ error.

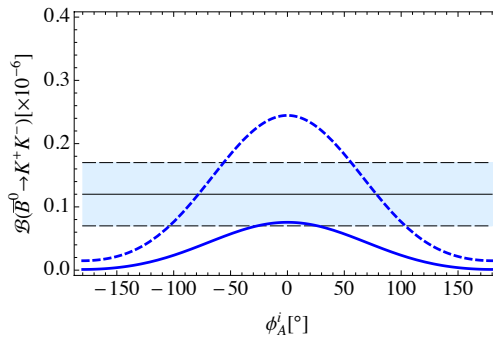


FIG. 3: The dependence of branching ratio $\mathcal{B}(\bar{B}^0 \rightarrow K^- K^+)$ on nonfactorizable annihilation parameters (ρ_A^i, ϕ_A^i) . The notes are the same as Fig.2.

To get more information on annihilation and spectator scattering, we perform a fit on the parameters $X_H = X_A^i$ and X_A^f , considering the constraints of the CP -averaged branch-

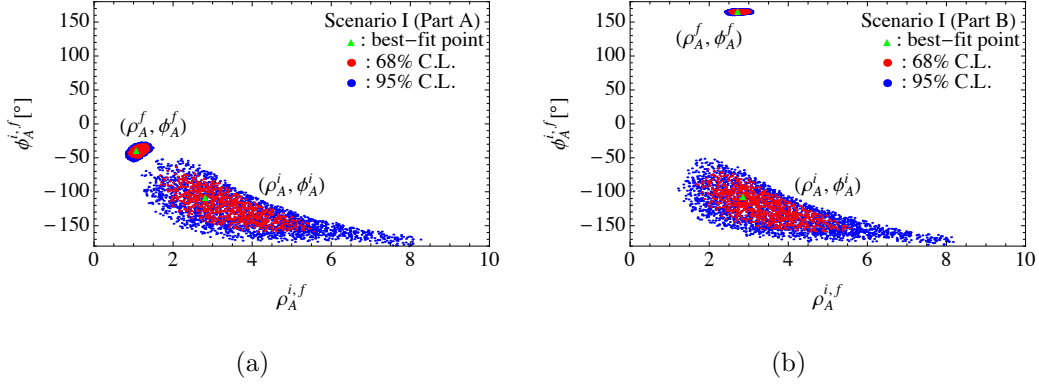


FIG. 4: The allowed regions of annihilation parameters at 68% C. L. and 95% C. L. in $(\rho_A^{i,f}, \phi_A^{i,f})$ planes, where the best-fit points of part A and B correspond to $\chi_{\min}^2 = 2.47$ and $\chi_{\min}^2 = 2.46$, respectively.

TABLE I: Numerical results of annihilation parameters in scenario I.

	$\rho_H = \rho_A^i$	$\phi_H = \phi_A^i [^\circ]$	ρ_A^f	$\phi_A^f [^\circ]$
Part A	$2.82_{-1.15}^{+2.73}$	-108_{-50}^{+44}	$1.07_{-0.20}^{+0.30}$	-40_{-11}^{+10}
Part B	$2.86_{-1.20}^{+2.68}$	-108_{-51}^{+42}	$2.72_{-0.22}^{+0.30}$	166_{-4}^{+3}

ing ratios, direct and mixing-induced CP -asymmetries, from $B \rightarrow \pi K$, $K\bar{K}$ decays. The experimental data are summarized in the second column of Tables II-IV. Our fitting results are shown by Fig.4, and the corresponding numerical results are listed in Table I-IV.

It is found that two possible solutions entitled Part A and B in Table I, correspond to almost the same $(\rho_A^i, \phi_A^i) \approx (2.8, -108^\circ)$. The large errors on parameter (ρ_A^i, ϕ_A^i) are mainly caused by the current loose experimental constraints on CP asymmetries measurements for $B \rightarrow \pi K$, $K\bar{K}$ decays. In principle, the pure annihilation $\bar{B}^0 \rightarrow K^- K^+$ decays whose amplitudes depend predominantly on (ρ_A^i, ϕ_A^i) , besides the decays constants, should give rigorous constraint on X_A^i . It's a pity that the available measurement accuracy on its branching ratio is too poor to efficiently confine (ρ_A^i, ϕ_A^i) to some tiny spaces. The large (ρ_A^i, ϕ_A^i) mean large X_A^i and X_H , i.e., there must exist large nonfactorizable annihilation and spectator scattering contributions to accommodate the current measurements. Our fit results on parameter ρ_A^i provide a robust evidence to the educated guesswork about $\rho_{Ad}^i = 2.5$ in Ref.[5]. In fact, the strong phase ϕ_A^i educed from measurements of branching ratios for

TABLE II: The CP-averaged branching ratios (in units of 10^{-6}) of $B \rightarrow \pi K, K\bar{K}, \pi\pi$ decays. For the Part A results of scenario I and II, the first and second theoretical uncertainties are caused by the CKM and other input parameters, respectively.

Decay Mode	Exp. [3]	scenario I	scenario II	S4 [11]
$B^- \rightarrow \pi^- \bar{K}^0$	23.79 ± 0.75	$20.53^{+1.52+4.28}_{-0.65-3.87}$	$21.54^{+1.60+4.40}_{-0.68-3.99}$	20.3
$B^- \rightarrow \pi^0 K^-$	$12.94^{+0.52}_{-0.51}$	$11.29^{+0.88+2.14}_{-0.45-1.96}$	$11.78^{+0.92+2.20}_{-0.47-2.01}$	11.7
$\bar{B}^0 \rightarrow \pi^+ K^-$	$19.57^{+0.53}_{-0.52}$	$17.54^{+1.34+3.61}_{-0.65-3.27}$	$18.51^{+1.41+3.73}_{-0.67-3.38}$	18.4
$\bar{B}^0 \rightarrow \pi^0 \bar{K}^0$	9.93 ± 0.49	$8.05^{+0.60+1.84}_{-0.27-1.65}$	$8.60^{+0.65+1.90}_{-0.29-1.72}$	8.0
$B^- \rightarrow K^- K^0$	1.19 ± 0.18	$1.45^{+0.13+0.32}_{-0.09-0.29}$	$1.51^{+0.13+0.32}_{-0.09-0.29}$	1.46
$\bar{B}^0 \rightarrow K^- K^+$	0.12 ± 0.05	$0.13^{+0.01+0.02}_{-0.01-0.02}$	$0.15^{+0.02+0.02}_{-0.01-0.02}$	0.07
$\bar{B}^0 \rightarrow K^0 \bar{K}^0$	1.21 ± 0.16	$1.22^{+0.11+0.27}_{-0.08-0.24}$	$1.32^{+0.12+0.27}_{-0.08-0.25}$	1.58
$B^- \rightarrow \pi^- \pi^0$	$5.48^{+0.35}_{-0.34}$	$5.20^{+0.64+1.11}_{-0.47-1.00}$	$5.59^{+0.68+1.15}_{-0.51-1.04}$	5.1
$\bar{B}^0 \rightarrow \pi^+ \pi^-$	5.10 ± 0.19	$5.88^{+0.66+1.66}_{-0.49-1.45}$	$5.74^{+0.64+1.63}_{-0.47-1.42}$	5.2
$\bar{B}^0 \rightarrow \pi^0 \pi^0$	$1.91^{+0.22}_{-0.23}$	$1.67^{+0.22+0.25}_{-0.19-0.23}$	$2.13^{+0.29+0.32}_{-0.24-0.29}$	0.7
$R_{+-}^{\pi\pi}$	1.99 ± 0.15	$1.64^{+0.06+0.13}_{-0.06-0.11}$	$1.80^{+0.07+0.17}_{-0.07-0.13}$	1.82
$R_{00}^{\pi\pi}$	0.75 ± 0.09	$0.57^{+0.06+0.16}_{-0.06-0.12}$	$0.74^{+0.08+0.22}_{-0.08-0.17}$	0.27

$B^0 \rightarrow K\bar{K}$ decays in Ref.[5] can have either positive or negative values with the magnitudes of $\gtrsim 100^\circ$ (see Fig.5 of Ref.[5]), where the positive value $\phi_A^i = +100^\circ$ used in Ref.[5] will be excluded by our fit with much more experimental data on $B \rightarrow \pi K, K\bar{K}$ decays. The large value of ϕ_A^i , corresponding to a large imaginary part of the enhanced complex corrections, also lends some support to the pQCD claim that the annihilation amplitudes can provide a large strong phase [8].

There are two possible solutions for the factorizable annihilation parameters, namely, Part A $(\rho_A^f, \phi_A^f) \approx (1.1, -40^\circ)$ and Part B $(\rho_A^f, \phi_A^f) \approx (2.7, 166^\circ)$. From Fig.4, it can be seen that there is no overlap between the regions of (ρ_A^f, ϕ_A^f) and (ρ_A^i, ϕ_A^i) at the 95% confidence level, which indicates that it might be wrong to treat $(\rho_A^f, \phi_A^f) = (\rho_A^i, \phi_A^i) = (\rho_A, \phi_A)$ as universal parameters for nonfactorizable and factorizable annihilation topologies in pervious studies. Our fit results certify the suggestion of Ref.[4, 5] that different annihilation topologies should be parameterized by different annihilation parameters, i.e., $(\rho_A^f, \phi_A^f) \neq (\rho_A^i, \phi_A^i)$. Compared with the results of (ρ_A^i, ϕ_A^i) , the errors on parameter (ρ_A^f, ϕ_A^f) are relatively small (see Table

TABLE III: The direct CP asymmetries (in units of 10^{-2}) of $B \rightarrow \pi K, K\bar{K}, \pi\pi$ decays. The notes on uncertainties are the same as TableII.

Decay Mode	Exp. [3]	scenario I	scenario II	S4 [11]
$B^- \rightarrow \pi^- \bar{K}^0$	-1.5 ± 1.9	$-0.05^{+0.00+0.13}_{-0.00-0.15}$	$-0.17^{+0.01+0.14}_{-0.01-0.15}$	0.3
$B^- \rightarrow \pi^0 K^-$	4.0 ± 2.1	$3.2^{+0.2+0.6}_{-0.2-0.6}$	$2.5^{+0.1+0.6}_{-0.1-0.6}$	-3.6
$\bar{B}^0 \rightarrow \pi^+ K^-$	-8.2 ± 0.6	$-7.7^{+0.4+0.9}_{-0.4-0.9}$	$-9.1^{+0.4+0.9}_{-0.5-0.9}$	-4.1
$\bar{B}^0 \rightarrow \pi^0 \bar{K}^0$	-1 ± 10	$-10.3^{+0.6+0.9}_{-0.6-1.0}$	$-10.6^{+0.6+0.9}_{-0.6-0.9}$	0.8
ΔA	12.2 ± 2.2	$10.9^{+0.6+0.9}_{-0.5-0.8}$	$11.6^{+0.6+0.9}_{-0.6-0.8}$	0.5
$B^- \rightarrow K^- K^0$	3.9 ± 14.1	$-0.6^{+0.0+3.2}_{-0.0-2.9}$	$2.0^{+0.1+3.4}_{-0.1-3.0}$	-4.3
$\bar{B}^0 \rightarrow K^0 \bar{K}^0$	-6 ± 26	-17^{+1+2}_{-1-2}	-16^{+1+2}_{-1-2}	-11.5
$B^- \rightarrow \pi^- \pi^0$	2.6 ± 3.9	$-1.1^{+0.1+0.1}_{-0.1-0.1}$	$-1.2^{+0.1+0.1}_{-0.1-0.1}$	-0.02
$\bar{B}^0 \rightarrow \pi^+ \pi^-$	29 ± 5	19^{+1+4}_{-1-4}	24^{+2+5}_{-2-4}	10.3
$\bar{B}^0 \rightarrow \pi^0 \pi^0$	43 ± 24	46^{+3+6}_{+3-6}	38^{+2+6}_{-2-6}	-19.0

TABLE IV: The mixing-induced CP asymmetries (in units of 10^{-2}) of $B \rightarrow \pi K, K\bar{K}, \pi\pi$ decays. The notes on uncertainties are the same as TableII.

Decay Mode	Exp. [3]	scenario I	scenario II
$\bar{B}^0 \rightarrow \pi^0 \bar{K}^0$	57 ± 17	78^{+3+1}_{-3-1}	79^{+3+1}_{-3-1}
$\bar{B}^0 \rightarrow K^- K^+$	—	-86^{+6+0}_{-5-0}	-86^{+6+0}_{-5-0}
$\bar{B}^0 \rightarrow K^0 \bar{K}^0$	-108 ± 49	-10^{+1+0}_{-1-0}	-11^{+1+0}_{-1-0}
$\bar{B}^0 \rightarrow \pi^+ \pi^-$	-65 ± 6	-59^{+11+2}_{-10-3}	-60^{+10+2}_{-10-2}
$\bar{B}^0 \rightarrow \pi^0 \pi^0$	—	77^{+6+1}_{-8-2}	77^{+7+1}_{-9-2}

I), because the available measurements on branching ratios for $B \rightarrow \pi K$ decays are highly precise. The conjecture about (ρ_A^f, ϕ_A^f) in [5] is somewhat alike to our fit results of Part A.

The value of term $(2X_A^f - X_A^f)$ in Eq.(A5) is about $(27.2 - i26.2)$ with parameters for Part A and $(28.9 - i25.5)$ for Part B, that is to say, these two solutions, Part A and B, will present similar factorizable annihilation contributions. Nevertheless, a small value of ρ_A^f is more easily accepted by the QCDF approach [11]. So with the best fit parameters of Part A in Table I, we present our evaluations on branching ratios, direct and mixing-

induced CP asymmetries for $B_{u,d} \rightarrow \pi K$, $K\bar{K}$, $\pi\pi$ decays in the “scenario I” column of Table II, III and IV, respectively. For comparison, the results of scenario S4 QCDF [11] are also collected in the “S4” column. It is easily found that all theoretical results are in good agreement with experimental data within errors. Especially, the difference ΔA , which $\sim 0.5\%$ in scenario S4 QCDF, is enhanced to the experimental level $\sim 11\%$. It is interesting that although $B \rightarrow \pi\pi$ decays are not considered in the “scenario I” fit, all predictions on these decays, including the ratios $R_{+-}^{\pi\pi}$ and $R_{00}^{\pi\pi}$, are also in good consistence with the experimental measurements within errors, which implies that the πK and $\pi\pi$ puzzles could be resolved by annihilation and spectator corrections, at the same time, without violating the agreement of other observables. The reason will be excavated in Scenario II.

B. Scenario II

From Eq.(18), it is obviously found that the amplitude of $B^- \rightarrow \pi^- \pi^0$ decay is independent of annihilation contributions, and dominated by $\alpha_1 + \alpha_2$. Moreover, comparing Eq.(19) with Eq.(20), it is easily found that the annihilation contributions are almost helpless for $R_{00}^{\pi\pi}$ puzzle due to $\mathcal{A}_{B^0 \rightarrow \pi^+ \pi^-}^{\text{anni}} \simeq \mathcal{A}_{B^0 \rightarrow \pi^0 \pi^0}^{\text{anni}}$. So, the spectator scattering corrections, which play an important role in the color-suppressed coefficient α_2 [11, 14, 17], would be another important key for the good results of scenario I, especially for $B \rightarrow \pi\pi$ decays.

Within QCDF framework, besides X_H , the inverse moment λ_B of B wave function defined by Eq.(10) is another important quantity in evaluating the contributions of spectator scattering. Unfortunately, its value is hardly to be obtained reliably with theoretical methods until now, for instance 350 ± 150 MeV (200 MeV in scenario S2) in Ref.[11], 200_{-0}^{+250} MeV in Ref.[19] and 300 ± 100 MeV in Ref.[14], though QCD sum rule prefer 460 ± 110 MeV at the scale of 1 GeV [20]. Experimentally, the upper limit on parameter λ_B are set at the 90% C.L. via measurements on branching fraction of radiative leptonic $B \rightarrow \ell \bar{\nu}_\ell \gamma$ decay by BABAR collaboration, $\lambda_B > 669$ (591) MeV with different priors based on 232 million $B\bar{B}$ sample where the photon is not required to be sufficiently energetic in order not to sacrifice statistics [21], and $\lambda_B > 300$ MeV based on 465 million $B\bar{B}$ pairs [22]. Considering radiative and power corrections, an improved analysis is preformed in Ref.[18] with the conclusion that present BABAR measurements cannot put significant constrains on λ_B and that $\lambda_B > 115$ MeV from the experimental results [22]. Anyway, the study of hadronic B decays

favors a relative small value of $\lambda_B \approx 200$ MeV to achieve a satisfactory description of color-suppressed tree decay modes [23]. At the present time, the value of λ_B is still a point of controversy. In the following analysis and evaluations, we treat λ_B as a free parameter.

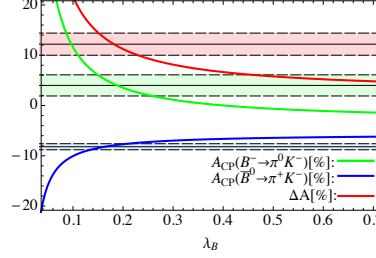


FIG. 5: The dependance of the direct CP asymmetries $A_{CP}(B^- \rightarrow \pi^0 K^-)$, $A_{CP}(\bar{B}^0 \rightarrow \pi^+ K^-)$ and their difference ΔA on λ_B (in unites of GeV) with the fitted annihilation parameters of scenario I (Part A). Their experimental results with 1σ error are shown by shaded bands with the same color as the lines.

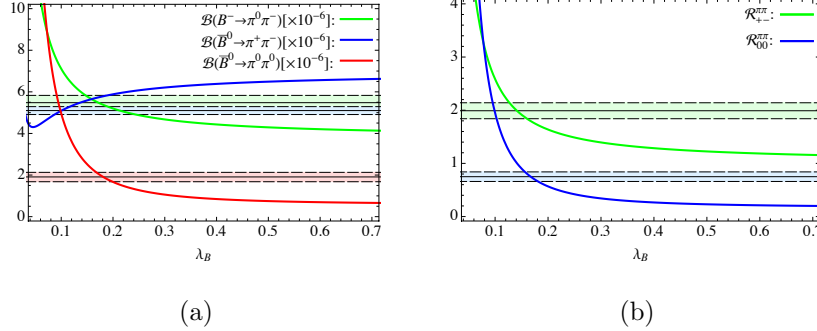


FIG. 6: The dependence of the branching fractions $\mathcal{B}(B^- \rightarrow \pi^- \pi^0)$, $\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \pi^-)$, $\mathcal{B}(\bar{B}^0 \rightarrow \pi^0 \pi^0)$ and ratios $R_{+-}^{\pi\pi}$, $R_{00}^{\pi\pi}$ on λ_B with the same notes as Fig.5.

TABLE V: Numerical results of annihilation parameters and moment parameter λ_B in Scenario II.

	ρ_A^i	$\phi_A^i [^\circ]$	ρ_A^f	$\phi_A^f [^\circ]$	λ_B [GeV]
Part A	$2.88^{+1.52}_{-1.30}$	-103^{+33}_{-40}	$1.21^{+0.22}_{-0.25}$	-40^{+12}_{-8}	$0.18^{+0.11}_{-0.08}$
Part B	$2.98^{+1.50}_{-1.40}$	-106^{+35}_{-39}	$2.78^{+0.29}_{-0.18}$	165^{+4}_{-3}	$0.19^{+0.09}_{-0.10}$

To explicitly show the effects of spectator scattering contributions on πK puzzle, dependance of $A_{CP}(B^- \rightarrow \pi^0 K^-)$, $A_{CP}(\bar{B}^0 \rightarrow \pi^+ K^-)$ and their difference ΔA on parameter λ_B

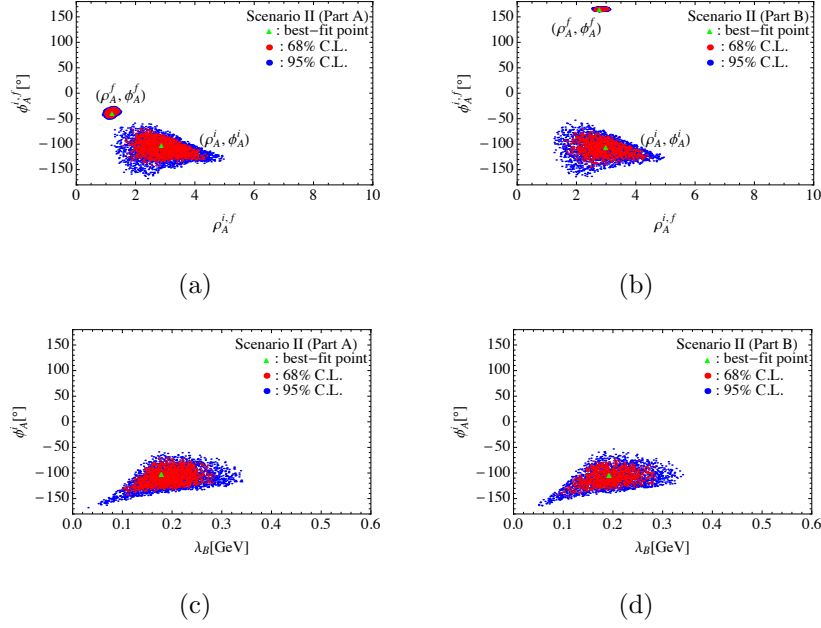


FIG. 7: The allowed regions of annihilation parameters $(\rho_A^{i,f}, \phi_A^{i,f})$ and λ_B at 68% C.L. and 95% C.L.. The best-fit points of part A and B correspond to $\chi_{\min}^2 = 3.66$ and $\chi_{\min}^2 = 3.67$, respectively.

are displayed in Fig.5. It is found that (1) observables of $A_{CP}(B^- \rightarrow \pi^0 K^-)$ and ΔA are more sensitive to variation of λ_B than $A_{CP}(\bar{B}^0 \rightarrow \pi^+ K^-)$ in the region of $\lambda_B \geq 100$ MeV. The reason is aforementioned fact that coefficient α_2 in amplitude $\mathcal{A}_{B^- \rightarrow \pi^0 K^-}$ [see Eq.(12)] receives significant spectator scattering corrections. A noticeable change of observables is easily seen in the low region of λ_B because spectator scattering corrections are inversely proportional to λ_B [see Eq.(10) and Eq.(A6)]. (2) a relative small value of $\lambda_B \in [150 \text{ MeV}, 220 \text{ MeV}]$, as expected in [23], is required to confront with available measurements. Especially, the value $\lambda_B \approx 190$ MeV provides a perfect description of the experimental data on $A_{CP}(B^- \rightarrow \pi^0 K^-)$, $A_{CP}(\bar{B}^0 \rightarrow \pi^+ K^-)$ and ΔA simultaneously. For $B \rightarrow \pi\pi$ decays, from Eqs.(18-20), it is easily seen that amplitude $\mathcal{A}_{B^- \rightarrow \pi^- \pi^0} \propto \alpha_1 + \alpha_2$, $\mathcal{A}_{\bar{B}^0 \rightarrow \pi^+ \pi^-} \propto \alpha_1$, $\mathcal{A}_{\bar{B}^0 \rightarrow \pi^0 \pi^0} \propto \alpha_2$. The coefficient α_2 , corresponding to the color-suppressed tree contribution, its value is small relative to α_1 , so the experimental data on $R_{+-}^{\pi\pi}$ can be well explained with scenario S4 QCDF where $X_A^i = X_A^f$ and $\rho_A^{f,i} = 1$ (see Table II). But as to observable $R_{00}^{\pi\pi}$ or/and branching ratio $\mathcal{B}(\bar{B}^0 \rightarrow \pi^0 \pi^0)$, an enhanced α_2 is desirable. Hence, the nonfactorizable spectator scattering contributions, which have significant effects on α_2 , would play an important role in studying the color-suppressed tree B decays, and possibly provide a

solution to the $\pi\pi$ puzzle. The dependencies of the branching fractions of $B \rightarrow \pi\pi$ decays and ratios $R_{+-}^{\pi\pi}$, $R_{00}^{\pi\pi}$ on λ_B are shown in Fig.6 where the fitted parameters of Part A in Table I is used. It is interesting that beside a large value ρ_H , a small value of $\lambda_B \sim 200$ MeV is also required to confront with experimental data on $\mathcal{B}(B \rightarrow \pi\pi)$, $R_{+-}^{\pi\pi}$ and $R_{00}^{\pi\pi}$.

With the available experimental data on $B \rightarrow \pi\pi$, πK and $K\bar{K}$ decays, we perform a comprehensive fit on both annihilation parameters $(\rho_A^{i,f}, \phi_A^{i,f})$ and B -meson wave function parameter λ_B . The allowed parameter spaces are shown in Fig.7, and the corresponding numerical results are summarized in Table V. Like scenario I, there are two allowed spaces which are labelled by part A and B. It is easily found that (1) parameters $(\rho_A^i, \phi_A^i) = (\rho_H, \phi_H)$ are still required to have large values (see Table V), that is to say, it is necessary for penguin-dominated or color-suppressed tree B decays to own large corrections from nonfactorizable annihilation and spectator scattering topologies. (2) There is still no overlap between the regions of (ρ_A^f, ϕ_A^f) and (ρ_A^i, ϕ_A^i) at the 95% confidence level. (3) The central values of $\rho_A^{i,f}$ are a little larger than those in scenario I. The uncertainties on (ρ_A^i, ϕ_A^i) are a little smaller than those in scenario I, because more processes from $B \rightarrow \pi\pi$ decays are considered in fitting and the amplitudes for $B \rightarrow \pi\pi$ decays are sensitive to X_A^i and X_H rather than X_A^f . (4) A small value of parameter $\lambda_B \leq 350$ MeV at the 95% confidence level is strongly required to reconcile discrepancies between results of QCDF approach and available experimental data on $B \rightarrow \pi\pi$, πK and $K\bar{K}$ decays.

The two solutions of scenario II, Part A and B, will give similar results, as discussed before. With the best fit parameters of Part A in Table V, we present our evaluations on branching ratios, direct and mixing-induced CP asymmetries for $B_{u,d} \rightarrow \pi K$, $K\bar{K}$, $\pi\pi$ decays in the “scenario II” column of Table II, III and IV, respectively. It is found that the central values of branching ratios for $B \rightarrow \pi\pi$, πK and $K\bar{K}$ decays, expect $\bar{B}^0 \rightarrow \pi^+\pi^-$ decay, with the Part A parameters of scenario II, are a little larger than those of scenario I (see Table II), because a bit larger values of $\rho_A^{i,f}$ and a bit smaller value of λ_B than those of scenario I are taken in scenario II. Compared with results of scenario S4 QCDF, agreement between theoretical results within two scenarios and experimental measurements is improved, especially for the observables ΔA , $R_{00}^{\pi\pi}$ and $A_{CP}(B^0 \rightarrow \pi\pi)$.

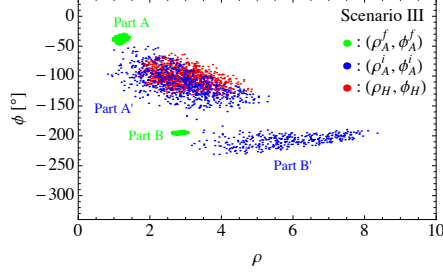


FIG. 8: The allowed regions of annihilation and hard-spectator parameters (ρ_A^f, ϕ_A^f) , (ρ_A^i, ϕ_A^i) and (ρ_H, ϕ_H) at 68% C.L.. The two solutions of (ρ_A^f, ϕ_A^f) and (ρ_A^i, ϕ_A^i) are labeled as Part A, B and A', B', respectively.

C. Scenario III

The above analyses and results are based on the assumption that $X_A^i = X_H$ (i.e. $(\rho_A^i, \phi_A^i) = (\rho_H, \phi_H)$) for simplicity. While, there is no compelling requirement for such simplification, except for the fact that wave functions of B mesons are involved in the convolution integrals of both spectator scattering and nonfactorizable annihilation corrections, but are irrelevant to the factorable annihilation amplitudes. So, as a general scenario (named scenario III), we would reevaluate the strength of annihilation and hard-spectator contributions without any simplification for the parameters (ρ_A^i, ϕ_A^i) , (ρ_A^f, ϕ_A^f) and (ρ_H, ϕ_H) .

Considering the constraints from observables of $B_{u,d} \rightarrow K\bar{K}$, πK and $\pi\pi$ decays, a fit for the annihilation and hard-spectator parameters is performed again. In this fit, (ρ_A^f, ϕ_A^f) , (ρ_A^i, ϕ_A^i) and (ρ_H, ϕ_H) are treated as six free parameters. Moreover, from the hard-spectator corrections illustrated by Eq. (A6), it can be seen that λ_B and X_H are always combined together.

Although the inverse moment λ_B of B wave function could be determined or constricted by further experiments [18, 21–23], λ_B is more like a free parameter for the moment due to loose limitation on it. So it is impossible to strictly bound on λ_B and X_H simultaneously due to the interference effects between them. In our following fit, we will fix $\lambda_B = 200$ MeV. Our fitting results at 68% C.L. are presented in Fig. 8, where the range of $\phi \in [-360^\circ, 0^\circ]$ is assigned to illustrate their relative magnitude. Numerically, we get

$$(\rho_A^f, \phi_A^f[^\circ]) = \begin{cases} (1.18_{-0.23}^{+0.26}, -40_{-8}^{+12}) & \text{Part A} \\ (2.79_{-0.20}^{+0.26}, -196_{-3}^{+5}) & \text{Part B} \end{cases} \quad (27)$$

$$(\rho_A^i, \phi_A^i[^\circ]) = \begin{cases} (2.85_{-1.92}^{+2.18}, -103_{-63}^{+52}) & \text{Part A'} \\ (6.54_{-3.30}^{+1.81}, -206_{-24}^{+23}) & \text{Part B'} \end{cases} \quad (28)$$

$$(\rho_H, \phi_H[^\circ]) = (3.09_{-1.53}^{+1.64}, -102_{-31}^{+40}). \quad (29)$$

It can be easily seen from Fig. 8 that: (1) for factorizable annihilation parameters (ρ_A^f, ϕ_A^f) , similar to scenarios I and II, there are two allowed regions (labelled by part A and B); (2) for nonfactorizable annihilation parameters (ρ_A^i, ϕ_A^i) , besides the solution similar to scenarios I and II (labelled by part A'), another solution (labelled by part B') with a very large value of ρ_A^i is gotten. (3) It is very interesting that the allowed space of (ρ_H, ϕ_H) overlaps almost entirely with the “part A'” allowed space of (ρ_A^i, ϕ_A^i) . Moreover, their best-fit points $(\rho_A^i, \phi_A^i) = (2.85, -103^\circ)$ of “part A'” and $(\rho_H, \phi_H) = (3.09, -102^\circ)$ are very close to each other. It might imply that the assumption $X_A^i(\rho_A^i, \phi_A^i) = X_H(\rho_H, \phi_H)$ used in scenarios I and II is a good simplification.

With the best fit parameters in scenarios III, either the small value of ρ_A^i in “part A'” or the large value in “part B'”, our evaluations on branching ratios, direct and mixing-induced CP asymmetries for $B_{u,d} \rightarrow \pi K, K\bar{K}, \pi\pi$ decays are similar to those given in our scenarios I and II, so no longer listed here. For the two solutions A' and B' of (ρ_A^i, ϕ_A^i) , it is expected by QCDF approach [11] that the parameter ρ_A^i should have a small value, which is also favored by our scenarios I and II fit. In fact, such two solutions lead to the same results of $A_{1,2}^i$, but the different ones of A_3^i , which principally provides an opportunity to refute one of them. However, because A_3^i is numerically trivial due to $(r_\chi^{M_1} - r_\chi^{M_2}) \sim 0$ for the light mesons, such way is practically unfeasible for current accuracies of theoretical calculation and experimentally measurement.

IV. CONCLUSIONS

The recent CDF and LHCb measurements of large branching ratios for pure annihilation $\bar{B}_s^0 \rightarrow \pi^+\pi^-$ and $\bar{B}_d^0 \rightarrow K^+K^-$ decays imply possible large annihilation contributions, which induce us to modify the traditional QCDF treatment for annihilation parameters. Following the suggestion of Ref.[5], two sets of annihilation parameters X_A^i and X_A^f are used to parameterize the endpoint singularity in nonfactorizable and factorizable annihilation amplitudes, respectively. Besides annihilation effects, the resolution of so-called πK and $\pi\pi$ puzzles also expect constructive contributions from spectator scattering topologies. With

the approximation of $X_A^i = X_H$, we perform a global fit on both annihilation parameters $(\rho_A^{i,f}, \phi_A^{i,f})$ and B -meson wave function parameter λ_B based on available experimental data for $B \rightarrow \pi\pi$, πK and $K\bar{K}$ decays. Our main conclusions and findings are summarized as:

- The 95% C.L. allowed region of (ρ_A^i, ϕ_A^i) is entirely different from that of (ρ_A^f, ϕ_A^f) . This fact means that the traditional QCDF treatment (ρ_A, ϕ_A) as universal parameters for different annihilation topologies might be unapplicable to hadronic B decays.
- The current experimental data on $B \rightarrow \pi\pi$, πK and $K\bar{K}$ decays seems to favor a large value of $\rho_A^i \sim 2.9$, which corresponds to a sizable nonfactorizable annihilation contributions. But the range of (ρ_A^i, ϕ_A^i) is still very large, because the measurement precision of CP asymmetries is low now.
- There are two possible choices for parameters (ρ_A^f, ϕ_A^f) . One is $(\rho_A^f, \phi_A^f) \sim (1.1, -40^\circ)$, the other is $(\rho_A^f, \phi_A^f) \sim (2.7, 165^\circ)$. These two choices correspond to similar factorizable annihilation contributions, although the QCDF approach tends to have a small value of ρ_A^f [11]. The space for (ρ_A^f, ϕ_A^f) is relatively tight due to the well measured branching ratios for $B \rightarrow \pi\pi$, πK and $K\bar{K}$ decays.
- The spectator scattering corrections play an important role in resolving both πK and $\pi\pi$ puzzles. Within QCDF approach, the spectator scattering amplitudes depend on parameters (ρ_H, ϕ_H) and B -meson wave function parameter λ_B . In our analysis, the approximation $(\rho_H, \phi_H) = (\rho_A^i, \phi_A^i)$ is assumed, which is proven to be a good simplification by a global fit in scenario III. A small value of $\lambda_B \leq 350$ MeV at the 95% C.L. is obtained by the global fit on $B \rightarrow \pi\pi$, πK and $K\bar{K}$ decays, which needs to be further tested by future improved measurement on $B \rightarrow \ell\nu_\ell\gamma$ decays. An enhanced color-suppressed tree coefficient α_2 , which is supported by both large value of $\rho_H \sim 2.9$ and small value of $\lambda_B \sim 200$ MeV, is helpful to reconcile discrepancies on ΔA and $R_{00}^{\pi\pi}$ between QCDF approach and experiments.

The spectator scattering and annihilation contributions can offer significant corrections to observables of hadronic B decays, and deserve intensive research especially when we apply the QCDF approach to the penguin-dominated, color-suppressed tree, and pure annihilation nonleptonic B decays. As suggested in Ref.[4, 5] and proofed by the pQCD approach

[8], different parameters corresponding to different topologies should be introduced to regulate the endpoint divergences in spectator scattering and annihilation amplitudes within QCDF approach, even parameters reflecting the flavor symmetry-breaking effects should be considered for $B_{u,d,s}$ decays [4–6, 11, 13, 14, 17]. This treatment might provide possible solution to “problematic” discrepancies between QCDF results and available measurements. Of course, a fine-tuning of these parameters is required to be compatible with the experimental constraints. With the running LHCb and the upcoming SuperKEKB experiments, more refined measurements on B -meson decays can be obtained, which will provide more powerful grounds to test various approach and confirm or refute some theoretical hypotheses.

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Appendix A: Building blocks of annihilation and spectator scattering contributions

The annihilation amplitudes for two-body nonleptonic $B \rightarrow M_1 M_2$ decays (here M_i denotes the light pseudoscalar meson) can be expressed as the following building blocks [11],

$$A_1^i = \pi\alpha_s \int_0^1 dx dy \left\{ \Phi_{M_2}^a(x) \Phi_{M_1}^a(y) \left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y} \right] + r_\chi^{M_1} r_\chi^{M_2} \frac{2\Phi_{M_2}^p(x) \Phi_{M_1}^p(y)}{\bar{x}y} \right\}, \quad (\text{A1})$$

$$A_2^i = \pi\alpha_s \int_0^1 dx dy \left\{ \Phi_{M_2}^a(x) \Phi_{M_1}^a(y) \left[\frac{1}{\bar{x}(1-x\bar{y})} + \frac{1}{\bar{x}y^2} \right] + r_\chi^{M_1} r_\chi^{M_2} \frac{2\Phi_{M_2}^p(x) \Phi_{M_1}^p(y)}{\bar{x}y} \right\}, \quad (\text{A2})$$

$$A_3^i = \pi\alpha_s \int_0^1 dx dy \left\{ r_\chi^{M_1} \frac{2\bar{y}}{\bar{x}y(1-x\bar{y})} \Phi_{M_2}^a(x) \Phi_{M_1}^p(y) - r_\chi^{M_2} \frac{2x}{\bar{x}y(1-x\bar{y})} \Phi_{M_1}^a(y) \Phi_{M_2}^p(x) \right\}, \quad (\text{A3})$$

$$A_1^f = A_2^f = 0, \quad (\text{A4})$$

$$A_3^f = \pi\alpha_s \int_0^1 dx dy \left\{ r_\chi^{M_1} \frac{2(1+\bar{x})}{\bar{x}^2 y} \Phi_{M_2}^a(x) \Phi_{M_1}^p(y) + r_\chi^{M_2} \frac{2(1+y)}{\bar{x}y^2} \Phi_{M_1}^a(y) \Phi_{M_2}^p(x) \right\}, \quad (\text{A5})$$

where the subscripts k on $A_k^{i,f}$ correspond to three possible Dirac current structures, namely, $k = 1, 2, 3$ for $(V - A) \otimes (V - A)$, $(V - A) \otimes (V + A)$, $-2(S - P) \otimes (S + P)$, respectively.

$r_\chi^M = 2m_M^2/m_b(m_1 + m_2)$, where $m_{1,2}$ are the current quark mass of the pseudoscalar meson with mass m_M . Φ_M^a and Φ_M^p are the twist-2 and twist-3 light-cone distribution amplitudes, respectively. Their asymptotic forms are $\Phi_M^a(x) = 6x\bar{x}$ and $\Phi_M^p(x) = 1$.

The spectator scattering corrections are given by [11]

$$H_i(M_1 M_2) = \begin{cases} +\frac{B_{M_1 M_2}}{A_{M_1 M_2}} \int_0^1 d\xi \frac{\Phi_B(\xi)}{\xi} \int_0^1 dx dy \left[\frac{\Phi_{M_2}^a(x) \Phi_{M_1}^a(y)}{\bar{x}\bar{y}} + r_\chi^{M_1} \frac{\Phi_{M_2}^a(x) \Phi_{M_1}^p(y)}{x\bar{y}} \right], \\ \quad \text{for } i = 1, 2, 3, 4, 9, 10 \\ -\frac{B_{M_1 M_2}}{A_{M_1 M_2}} \int_0^1 d\xi \frac{\Phi_B(\xi)}{\xi} \int_0^1 dx dy \left[\frac{\Phi_{M_2}^a(x) \Phi_{M_1}^a(y)}{x\bar{y}} + r_\chi^{M_1} \frac{\Phi_{M_2}^a(x) \Phi_{M_1}^p(y)}{\bar{x}\bar{y}} \right], \\ \quad \text{for } i = 5, 7 \\ 0, \quad \text{for } i = 6, 8 \end{cases} \quad (\text{A6})$$

where the factorized matrix elements are parameterized as [11]

$$A_{M_1 M_2} = i \frac{G_F}{\sqrt{2}} m_B^2 F_0^{B \rightarrow M_1} f_{M_2}, \quad B_{M_1 M_2} = i \frac{G_F}{\sqrt{2}} f_B f_{M_1} f_{M_2}. \quad (\text{A7})$$

Appendix B: Theoretical input parameters

For the CKM matrix elements, we adopt the fitting results for the Wolfenstein parameters given by the CKMfitter group [24]

$$\bar{\rho} = 0.140_{-0.026}^{+0.027}, \quad \bar{\eta} = 0.343_{-0.014}^{+0.015}, \quad A = 0.802_{-0.011}^{+0.029}, \quad \lambda = 0.22543_{-0.00094}^{+0.00059}. \quad (\text{B1})$$

The pole masses of quarks are [25]

$$\begin{aligned} m_u = m_d = m_s &= 0, \quad m_c = 1.67 \pm 0.07 \text{ GeV}, \\ m_b &= 4.78 \pm 0.06 \text{ GeV}, \quad m_t = 173.5 \pm 1.0 \text{ GeV}. \end{aligned} \quad (\text{B2})$$

The running masses of quarks are [25]

$$\begin{aligned} \frac{\bar{m}_s(\mu)}{\bar{m}_q(\mu)} &= 27 \pm 1, \quad \bar{m}_s(2 \text{ GeV}) = 95 \pm 5 \text{ MeV}, \quad \bar{m}_c(\bar{m}_c) = 1.275 \pm 0.025 \text{ GeV}, \\ \bar{m}_b(\bar{m}_b) &= 4.18 \pm 0.03 \text{ GeV}, \quad \bar{m}_t(\bar{m}_t) = 160.0_{-4.3}^{+4.8} \text{ GeV}. \end{aligned} \quad (\text{B3})$$

The decay constants of B -meson and light mesons are [25]

$$f_B = (0.190 \pm 0.013) \text{ GeV}, \quad f_\pi = (130.4 \pm 0.2) \text{ MeV}, \quad f_K = (156.1 \pm 0.8) \text{ MeV}. \quad (\text{B4})$$

We take the following heavy-to-light transition form factors [26]

$$F_0^{B \rightarrow \pi}(0) = 0.258 \pm 0.031, \quad F_0^{B \rightarrow K}(0) = 0.331 \pm 0.041. \quad (\text{B5})$$

Moreover, for the Gegenbauer coefficients, we take [27]

$$a_1^\pi(2\text{GeV}) = 0, \quad a_2^\pi(2\text{GeV}) = 0.17, \quad a_1^K(2\text{GeV}) = 0.05, \quad a_2^K(2\text{GeV}) = 0.17. \quad (\text{B6})$$

For the other inputs, such as the masses and lifetimes of mesons and so on, we take their central values given by PDG [25].

Appendix C: Fitting Approach

Our fit is performed in a simple way, which is similar to the one adopted in Ref.[28] based on the frequentist framework. Considering a set of N observables f_j , the experimental measurements are assumed to be Gaussian distributed with the mean value $f_{j\text{exp}}$ and error $\sigma_{j\text{exp}}$. The theoretical prediction $f_{j\text{theo}}$ for each observable could be treated as a function of a set of “unknown” free parameters $\{y_i\}$ (here $y_i = \rho_A^{i,f}, \phi_A^{i,f}$ and λ_B in this paper). To estimate the values of “unknown” parameters $\{y_i\}$ and compare the theoretical results $f_{j\text{theo}}$ with the experimental data $f_{j\text{exp}}$, typically, it is need to construct a χ^2 function as

$$\chi^2(\{y_i\}) = \sum_{j=1}^N \frac{(f_{j\text{theo}}(\{y_i\}) - f_{j\text{exp}})^2}{\sigma_{j\text{exp}}^2}. \quad (\text{C1})$$

In the evaluation of $f_{j\text{theo}}$ for hadronic B decays, ones always encounter theoretical uncertainties induced by input parameters, like form factor and decay constant, whose probability distribution is unknown. Following the treatment of Rfit scheme [24, 29] that input values are treated on an equal footing, irrespective of how close they are from the edge of the allowed range, the χ^2 function is modified as [28]

$$\chi^2 = \sum_{j=1}^N \begin{cases} \frac{([f_{j\text{theo}} - \delta_{j\text{theo,sub}}] - f_{j\text{exp}})^2}{\sigma_{j\text{exp}}^2} & \text{if } f_{j\text{exp}} < [f_{j\text{theo}} - \delta_{j\text{theo,sub}}], \\ \frac{(f_{j\text{exp}} - [f_{j\text{theo}} + \delta_{j\text{theo,sup}}])^2}{\sigma_{j\text{exp}}^2} & \text{if } f_{j\text{exp}} > [f_{j\text{theo}} + \delta_{j\text{theo,sup}}], \\ 0 & \text{otherwise} \end{cases} \quad (\text{C2})$$

where $\delta_{j\text{theo,sup}}$ and $\delta_{j\text{theo,sub}}$ denote asymmetric theoretical uncertainties, and are defined as $(f_{j\text{theo}})_{-\delta_{j\text{theo,sub}}}^{+\delta_{j\text{theo,sup}}}$. As to the asymmetric experimental errors, we choose the larger one as

weighting factor. Correspondingly, the confidence levels are defined by the function

$$\text{CL}(\{y_i\}) = \frac{1}{\sqrt{2^{N_{\text{dof}}}} \Gamma(N_{\text{dof}}/2)} \int_{\Delta\chi^2(\{y_i\})}^{\infty} e^{-t/2} t^{N_{\text{dof}}/2-1} dt, \quad (\text{C3})$$

with $\Delta\chi^2 = \chi^2 - \chi_{\text{min}}^2$ and N_{dof} the number of degrees of freedom of free parameters.

With the input parameters summarized in Appendix B, we scan the space of the parameters y_i and calculate the theoretical results $f_{j\text{theo}}$. The χ^2 could be obtained with Eq.(C2). The numerical results at 1σ and 2σ confidence levels are gotten from Eq.(C3) by taking $\text{CL} = 1 - 68.27\%$ and $\text{CL} = 1 - 95.45\%$, respectively.

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